OPTICAL CHARACTERISTICS OF GAS-DYNAMIC PRISMS

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An aerooptical element, the gas-dynamic prism, is theoretically and experimentally studied. The gas-dynamic prism enables one to deviate a light beam by significant angles. A model for such a prism and a technique for calculating its gas-dynamic and optical characteristics are proposed. Good agreement is noted between the calculations and the experimental data.

When solving a number of engineering problems in applied optics, it is useful to use gas prisms instead of their solid state analogues when deflecting light beams. However, the thermooptical gas prisms developed to date, having completely refractive characteristics for small angles of light beam deviation, become quite ineffective at large deflection angles. The cause of this is that thermal effects do not produce the required gradient in the index of refraction and, consequently, it is necessary to increase the size of the device to produce large deflection angles. However, due to the laminar flow losing stability, this means cannot be physically realized. In this connection, there is definite interest in the gas-dynamic prism, the optical nonuniformity in which is created by accelerating a supersonic gas flow. Here, significant density gradients, and consequently refractive index gradients, can be attained with a sharp pressure drop.

A one-dimensional model of a gas-dynamic prism was examined for the first time theoretically by Christiansen [1], which was subsequently verified experimentally in [2, 3]. However, for a light beam with a finite aperture, the one-dimensional approximation does not reflect the real distribution pattern of the light parameters. By virtue of the hydrodynamic features of the flow in a Laval nozzle, the distribution of the refractive index does not provide the optimum conditions for passing light in all regions of the flow. As a result, undesired distortions arise in the wavefront of the transmitted radiation.

This work cites the results of theoretical and experimental studies of the parameters of a gas-dynamic prism, the model for which is a specially profiled planar Laval nozzle with a linear refractive index distribution along the nozzle's symmetry axis. Possible variations in the propagation direction of the transmitted wavefront are analyzed while allowing for spatial nonuniformities in the distribution of gas-dynamic and optical parameters in the gaseous medium. Using the method of characteristics, the gas-dynamic parameters in the supersonic flow region were calculated and the nozzle contour was plotted [4]. To do this, the inverse problem was solved for a planar Laval nozzle with a given linear refractive index distribution along the nozzle's symmetry axis, taken to be the OX-axis of a rectangular Cartesian coordinate system (see Fig. 1). The flow was ensured to be shockless by matching the supersonic flow region with the solution in the subsonic and transonic parts of the nozzle, derived in [5].

The refractive index of the medium is expressed in terms of the density via the relation

$$n = 1 + \beta \frac{\rho}{\rho_0}, \tag{1}$$

where β is the Gladstone–Dale constant (for air $\beta = 2.92 \cdot 10^{-4}$), ρ is density of the medium, and ρ_0 is the adiabatically retarded gas density. The solution to the problem of the propagation of light in a gas-dynamic prism was examined in the geometric optics approximation. Here we used the equation for the trajectory of a ray in the form

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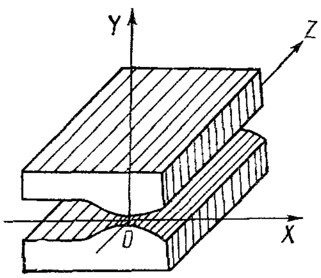

Fig. 1. Model of the gas-dynamic prism and the coordinate system.

$$\frac{d^2\mathbf{r}}{dS^2} = \operatorname{grad} n, \tag{2}$$

where the parameter S is related to the element of arc σ by the relation:

$$S = \int_0^\sigma \frac{d\sigma}{\sqrt{n}}, \quad d\sigma = \sqrt{dx^2 + dy^2 + dz^2}.$$

A solution was sought in analytical form. To do this, according to calculated data and by virtue of the monotonic nature of the distribution of the magnitude of n, the refractive index field can be approximated to a sufficient degree by the expression

$$n(x, y) = a - bx - cy^2. \tag{3}$$

Here x and y are dimensionless coordinates, referenced to half the height in the output cross section of the nozzle. The relative error of the approximation in all examined gases did not exceed 1%. The value of the coefficients a, b, c for several Mach numbers and $\rho_{out}/\rho^* = 0.2$ are given in Table 1. After substituting (3) into (2), we obtain the following simple form:

$$\frac{d^2x}{dS^2} = -b, \quad \frac{d^2y}{dS^2} = -2cy, \quad \frac{d^2z}{\partial S^2} = 0$$

and has the general solution suitable for further analysis:

$$x = -\frac{b}{2}S^2 + a_1 S + a_2,$$
$$y = b_1 \sin(S\sqrt{2c}) + b_2 \cos(S\sqrt{2c}), \quad z = SC_1 + C_2. \tag{4}$$

Further, if one assumes that the entrance to the gas-dynamic prism coincides with the OXY plane, then the ray's calculated trajectory σ will diverge away from this plane and then it is necessary to set $C_2 = 0$. The constant C_1 is expressed in terms of the direction cosine of the ray trajectory at the entrance:

$$C_1 = \sqrt{n_0} \cos(\widehat{\mathbf{z}_0, \boldsymbol{\sigma}_0}), \tag{5}$$

where σ_0 is the ray's unit propagation vector, n_0 is the refractive index at the entrance point, and z_0 is the unit vector along the Z-axis. Similarly, recalling the relation

$$\frac{d\mathbf{r}}{dS} = \sqrt{n}\frac{d\mathbf{r}}{d\sigma},$$

we obtain

$$a_1 = \sqrt{n_0} \cos(\widehat{\mathbf{x}_0, \boldsymbol{\sigma}_0}); \quad b_1 = \sqrt{\frac{n_0}{2c}} \cos(\widehat{\mathbf{y}_0, \boldsymbol{\sigma}_0});$$
$$a_2 = x_0; \quad b_2 = y_0. \tag{6}$$

TABLE 1. Values of the Approximation Coefficients for Several Mach Numbers

Mach number	a	b	c
2,0	1,001909	0,000707	0,000468
2,5	1,004046	0,001373	0,001157
3,0	1,008557	0,002824	0,002721

TABLE 2. Values of the Effective Lengths of the Gas-Dynamic Prisms and Exit Angles

ρ_{out}/ρ^*	Mach number, M	Launch coordinate of the ray x_0, mm	Effective length Z_ℓ, mm	Exit angle α, deg
0,2	2,0	1,73707	70,125	2,84
		1,047367	54,467	2,20
		0,357665	31,834	1,30
	3,0	2,41812	41,419	3,34
		1,11570	28,185	2,26
		0,33430	15,445	1,24

Now, using (5) and (6), the solution (4) finally acquires the form:

$$\begin{aligned} x &= -\frac{b}{2}S^2 + \sqrt{n_0}\cos(\widehat{x_0, \sigma_0})\,S + x_0; \\ y &= \sqrt{\frac{n_0}{2c}}\cos(\widehat{y_0, \sigma_0})\sin(S\sqrt{2c}) + y_0\cos(S\sqrt{2c}); \\ z &= \sqrt{n_0}\cos(\widehat{z_0, \sigma_0})\,S. \end{aligned} \tag{7}$$

If the ray is launched parallel to the OZ-axis, then $\cos(\widehat{x_0, \sigma_0}) = 0$, $\cos(\widehat{y_0, \sigma_0}) = 0$, and the equation of the trajectory takes the simpler form:

$$x = -\frac{b}{2}S^2 + x_0, \quad y = y_0\cos(S\sqrt{2c}), \quad z = \sqrt{n_0}S. \tag{8}$$

If the ray is not launched in the prism's plane of symmetry, then the spatial trajectory of the ray is composed of a parabolic displacement along the X-axis and a cosinusoidal trajectory along the Y-axis. The larger the Mach number is, the less time the ray spends in the gradient zone. We shall calculate the exit angle and the corresponding effective length of the prism Z_l for this ray.

From the first equation of system (8) it follows that for a prism with a Mach number M = 2.0 and $\rho_{out}/\rho^* = 0.2$, its length will be

$$Z_l = Z|_{\alpha=l} = \sqrt{x_0\frac{n(x_0)}{0{,}0003535}}, \tag{9}$$

and the exit angle

$$\alpha = \operatorname{arctg}\frac{dx}{dz} = -\operatorname{arctg}\left[\frac{0{,}000707}{n_0}Z_l\right]. \tag{10}$$

Here x_0 is the coordinate of the entering ray. The results of calculations by the formulas of the effective length and exit angle for two prisms are cited in Table 2. Note that if the ray is launched into the prism at x = 0 at the angle α, determined from Eq. (10), then according to the principle of reversibility of trajectories of light rays, it exits from the prism at the point z = 0, $x = x_0$ parallel to the Z-axis. Therefore, if the prism is extended in the region of negative values of z by the amount Z_l, the trajectory of the ray in this region of the prism, in correspondence with the duality principle, will be the second branch of a parabola that is symmetric to the input branch. Thus, by choosing

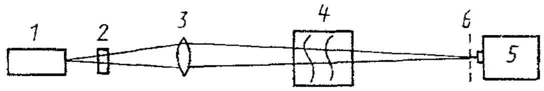

Fig. 2. Optical system for measuring the deflection angle of a light beam in a gas-dynamic prism: 1) LG-38 laser; 2) polarization filter; 3) long focal length lens; 4) prism under study; 5) RFK-5 camera without objective lens; 6) reference grid.

TABLE 3. Experimental Results of the Deflection of a Beam at Coordinate x_0 = 2.4 mm for a Prism Computed for Mach Number M = 3

n	1	2	3	4	5	6	7	8	9	10	11
α_i	3,06	3,11	2,94	3,0	2,86	2,89	2,92	3,12	2,81	2,76	2,97

the launch conditions of the ray in the manner indicated above, it is possible to increase by a factor of two the effective lengths of the prism cited in Table 2 and, correspondingly, to double the angle of rotation at each prism.

On the basis of the results of solving the inverse gas-dynamic problem, a series of planar nozzles were built and prepared for different Mach numbers and values of ρ_{out}/ρ^* lying in the interval 0.01-0.4.

To experimentally determine the angle of deflection of the gas-dynamic prisms, a prism was used that had a Mach number of M = 3. The launch coordinate of the ray x_0 was chosen from Table 2 to be x_0 = 2.4 mm, for which the predicted deflection angle was 3°20'.

The optical system for measuring the deflection angle of a laser beam by the prism is shown in Fig. 2. The light from an LG-38 He–Ne laser, attenuated by polarization filter 2, chosen to be type KN-2 film during precalibration, passes through long focal length lens 3, the gas-dynamic prism 4, and is focused on the film of an RFK-5 camera 5 without its objective lens. A grid 6 of vertically mounted wires 0.08 mm in diameter is placed just in front of the camera film. The principle of measuring the deflection angle is as follows. A reference laser beam, launched at coordinate x_0, was recorded on the film along with the shadow of one of the wires located on its path. The shadow from the wire after exposing the film was displayed in the form of bright band. On passing through an operating gas-dynamic prism, the ray was deflected in the direction of increasing flow density (toward the camera), which was also recorded on the film. The variance in the shifted reference and perturbed rays relative to the bright band, measured at the level of 0.5 of maximum blackening of the film from the reference and perturbed beams, gave the desired shift. The angle of deflection produced by the prism was determined from the formula:

$$\alpha = \operatorname{arctg}\frac{\Delta x}{L},$$

where Δx is the shift of the ray in the prism and L is the distance from the center of the prism to the film and was taken to be 76.5 mm. The gas parameters and the flow characteristics in the process of the experiment were maintained as constant as possible. The experimental technique and details are described in more detail in [6].

A series of eleven experiments was performed in which the ray's launch coordinate x_0 remained constant. The measurement data are cited in Table 3. The sources of random error during the measurement were the error related to the deviation of the thermodynamic parameters from their average values, the instability of the launch of the ray in the x_0 coordinate, etc. As a result of processing the data of Table 3 by the method of [7] it was established that the random errors while measuring the angles of deflection by the gas-dynamic prism were subject to a normal distribution.

We shall perform a confidence estimate and determine the magnitude of the random error ε for this series of experiments. Taking the Student relation $|\alpha_i - \alpha| < t(P, n)(s/\sqrt{n})$ and choosing a Student coefficient (0.95; 11) = 2.2 (confidence probability P = 0.95), we get $|\varepsilon| < 0.078°$.

Thus, it can be concluded that with a confidence probability of $P = 0.95$, the desired value of deflection angle from the gas-dynamic prism for a launch coordinate $x_0 = 2.4$ mm deviates from the measured average value $\alpha = 2.95°$ by an amount no greater than $0.078° = 4.7'$. The relative measurement error was 2.5%.

CONCLUSIONS

The theoretical and experimental studies conducted on the gas-dynamic prism show that it can be used more effectively than traditional aerooptical elements for deflecting a light beam at large angles. However, the degradation in the beam divergence of the light in the direction transverse to the axis of the nozzle requires one to perform some compensating measures to correct it. In the latter case, in all probability, the most suitable approach is to use a gas-dynamic system for beam deflection in combination with well-known thermal concentration and diffusion compensation methods in the transverse direction; this allows for the fact that the perturbation of the angles introduced by the prism in this direction are small.

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EXISTENCE OF STATIONARY WAVES OF RADIATION COOLING

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The solution of the system of equations of gas dynamics and radiation transfer is analyzed and it is shown that a Zel'dovich–Raizer stationary wave of radiation cooling does not exist in a hot gas.

The radiation cooling of a hot volume of air was studied in [1, 2]. It was shown that because of the extremely sharp temperature dependence of the optical properties of air such cooling must occur in the form of a temperature step propagating in the hot air – a so-called wave of cooling (WC) [3]. Cooling by radiation in this manner drops the air temperature from 10^5-10^6 K and higher to $\sim 10^4$ K within a short time. Assuming that the velocity of the wave of cooling is low compared with the velocity of sound, the authors neglected, owing to its smallness, the pressure jump on the front of the WC and the air motion arising, and instead of considering the complete system of equations of gas dynamics and radiation transfer, which describe this process, they limited their analysis to the energy equation and the radiation transfer equation. After integrating these equations,

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